CHAPTER 1

The Theory of Income Determination

The readings in this chapter are divided into two parts. Those in the first part deal with the determination of aggregate income, employment, and prices and the propagation of income changes, with special reference to the way in which fiscal and monetary policies may influence aggregate demand. The readings in the second part deal with the question of the inflationary process, its measurement and costs, and its relationship to employment.

A. Monetary and Fiscal Policies and Aggregate Demand

It is important for the student beginning the study of national income and money and banking to develop an integrated framework which he can use effectively to analyze the problems and issues that arise. It is best that he get this framework at the start so that the relevant institutional material can be fitted into it as he progresses. It is our hope that the material presented in this introduction, together with the readings contained in this section, will help the student to develop such a framework of analysis.

Economic reality is exceedingly complex, involving the outputs and prices of thousands of goods and services, the wages of thousands of different kinds of labor, and so on. If the economist tried to deal with all of the vast multitude of variables and relationships involved, he would soon become hopelessly bogged down. The only way to make headway, therefore, is to work with "models" which abstract from most of the detail and focus on the important variables related to the issue at hand. Of course, the model to be used depends on the kind of problem being dealt with. The models we shall develop have proved to be useful in analyzing the forces determining many of the major variables relating to the economy as a whole: the level of national income and employment, the general level of prices, etc. While we shall attempt to keep the models relatively simple, we feel that they represent the major economic relationships sufficiently well to enable the student who has a thorough grasp of them to comprehend and analyze many important issues of economic policy. It should be pointed out that there has been much statistical testing of models which, while more detailed and complex than those presented here, are of essentially the same character; and the statistical testing suggests that they explain the behavior of the economy quite well. Indeed, the results of some of the statistical studies are presented in readings included in this book.

We shall begin with the simplest kind of Keynesian static multiplier model of income determination with which the student is almost surely familiar from his other reading. Then we shall proceed to introduce fiscal and monetary elements in a way which, we hope, will help the student to understand questions of economic policy. We shall use an algebraic and arithmetic approach for the most part; however, the algebra does not extend beyond that covered in a course that would be taken in high school, or at the most in the first year of college. We shall also stick to linear relationships—that is, relationships that appear as straight lines when plotted graphically. Linear relationships are often reasonably good approximations to reality; moreover, the gain in simplicity of presentation is great.

Throughout the present discussion, no attention is paid to changes in the price level; in effect, we shall be assuming that prices (and wages) are unchanged and that changes in the money values of variables are paralleled by changes in their real values. However, the analysis is broadened in the first two readings in this section—the papers by Robert S. Holbrook and Warren L. Smith—which treat prices and wages as variables which are determined by the interplay of economic forces as are income, employment, etc.

The presentation in this introduction is divided into two major parts. The first deals with static analysis—that is, it is merely designed to tell what will ultimately happen to the variables when some change is introduced into the model, without making any effort to describe the time paths followed by the variables in the process of adjustment. The second part introduces some quite elementary dynamics.

1. STATIC ANALYSIS

Model I: The Simple Keynesian Multiplier

This model is represented by the following three algebraic equations:

$$C = C_o + cY \qquad \text{(consumption function)} \qquad (1.1)$$
$$I = I_o \qquad \text{(investment relationship)} \qquad (1.2)$$
$$Y = C + I \qquad \text{(equilibrium condition)} \qquad (1.3)$$

Here C is consumption expenditure planned by households, I is investment expenditure planned by firms, and Y is gross national product (GNP). The subscript o indicates that the variable is not explained within the model but is determined by outside forces. In this model C_0 stands for the amount of consumption which is unrelated to income, and c is the marginal propensity to consume (MPC), assumed to be a positive fraction between zero and unity in value. By substituting the expressions for C and I given by equations (1.1) and (1.2) into equation (1.3), we obtain the following:

$$Y = cY + C_0 + I_0 . \tag{1.4}$$

When this is solved for Y, the following result is obtained:

$$Y = \frac{1}{1-c}\,(C_0 + I_0) . \tag{1.5}$$

If there is a change in C_0 or I_0, income and consumption will also change. Suppose investment spending rises to a new level, $I_0 + \Delta I_0$, and remains there. Then we will find that income will also change by some amount, ΔY, so that the new level of income may be expressed as follows:

$$Y + \Delta Y = \frac{1}{1-c}\,(C_0 + I_0) + \frac{1}{1-c}\,\Delta I_0 . \tag{1.6}$$

Subtraction of (1.5) from (1.6) results in the following expression for the change of income (from the former equilibrium position to the new equilibrium) due to the change in investment spending:

$$\Delta Y = \frac{1}{1-c}\,\Delta I_0 . \tag{1.7}$$

Since ΔI_0 is multiplied by the term $1/(1-c)$ to obtain the income change, ΔY, this term is called the "multiplier." This is the standard textbook "formula" expressed in the statement, "the multiplier equals $1/(1-MPC)$." The student should not, however, view it as a formula to be memorized but rather as a relationship which summarizes a complex pattern of economic behavior, a pattern to be thought through and understood. It is particularly important to realize that the multiplier expression changes as the details of the model change, and that in the real world the multiplier process cannot be summarized in as simple a formula as that shown above. We shall now make the model, and the multiplier expression, somewhat more realistic.

Model II: The Introduction of Fiscal Policy

One of the most serious shortcomings of the simple model just discussed is that no allowance is made for the activities of government. To correct this defect we shall introduce government expenditures and taxation. For simplicity, we shall assume that all taxes are levied on households and that consumption depends on *disposable* income—that is, income after taxes. The new model is expressed in the following equations:

$$
\begin{aligned}
C &= C_o + cY_d \qquad 0 < c < 1 && (2.1)\\
Y_d &= Y - T && (2.2)\\
T &= T^* + xY \qquad 0 < x < 1 && (2.3)\\
I &= I_o && (2.4)\\
G &= G^* && (2.5)\\
Y &= C + I + G && (2.6)
\end{aligned}
$$

In this and following models, as was the case above, the subscript o identifies variables which are determined by forces outside of the model and which cannot be controlled by the government for policy purposes. We now introduce a second category of variables determined outside of the model: those which are manipulable by the authorities. Such variables are sometimes called "policy instruments" and will be denoted by an asterisk (*) throughout the remainder of this discussion. In Model II, government spending for goods and services (G^*) and that part of tax collections which is unrelated to income (T^*) are policy instruments. The equation $G = G^*$ states that the entire amount of government spending is determined outside of the model, while the equation describing tax collections, equation (2.3), indicates that only a part of total collections is under the direct control of the fiscal authorities. T is total collections, and it is composed of T^*, the level set by the authorities, plus xY, the part related to the level of income. The coefficient x is the marginal propensity of the public to pay taxes out of GNP.[1] Finally, Y_d is disposable income (i.e., household income after taxes); and c is the marginal propensity to consume out of disposable income.

Upon substitution of equations (2.2) and (2.3) into (2.1), the following equation is obtained:

$$C = C_o - cT^* + c(1-x)Y\,. \qquad (2.7)$$

Then, equations (2.4), (2.5), and (2.7) can be substituted into equation (2.6) to obtain

$$Y = C_o - cT^* + c(1-x)Y + I_o + G^*\,. \qquad (2.8)$$

Solving this equation explicitly for Y, we obtain

$$Y = \frac{1}{1-c(1-x)}[C_o - cT^* + I_o + G^*]\,. \qquad (2.9)$$

Suppose now that government purchases of goods and services are in-

[1]Strictly speaking x as well as T^* should be regarded as a policy instrument, since changes in tax legislation could (and, in practice, usually would) change the slope as well as the level of the tax function. In the interest of simplicity, however, we are confining our analysis to changes in the level of taxes (T^*).

creased from G^* to $G^* + \Delta G^*$. The new equilibrium income will be given by

$$Y + \Delta Y = \frac{1}{1-c(1-x)}[C_o - cT^* + I_o + G^* + \Delta G^*] \tag{2.10}$$

Subtracting (2.9) from (2.10) and dividing through by ΔG^*, we obtain the multiplier applicable to government purchases:

$$\frac{\Delta Y}{\Delta G^*} = \frac{1}{1-c(1-x)}. \tag{2.11}$$

Multipliers could also be computed for independent changes in investment (ΔI_o), in the level of consumption (ΔC_o), or in the level of taxes (ΔT^*). The first two of these multipliers would be the same as that for a change in government purchases while the multiplier for a change in taxes would be

$$\frac{\Delta Y}{\Delta T^*} = \frac{-c}{1-c(1-x)}.$$

This last multiplier is negative, because an increase in taxes would lower disposable income, reduce consumption, and lead to a decline in income.

There is a final technical point which should be noted. So far, all of the multipliers we have discussed have summarized the effects on GNP of a change in one of the variables determined by forces outside the model. The multiplier concept is more general than this, however, and it is possible to derive a multiplier expression which summarizes the effect of a shift in any of these variables on any variable determined within the model. Thus, for example, the effects on total tax collections (ΔT) of a shift in the level of the consumption function (ΔC_o) can easily be derived. From the tax function (2.3) we note that

$$\frac{\Delta T}{\Delta C_o} = x\frac{\Delta Y}{\Delta C_o}.$$

Using the approach employed in deriving the multiplier $\frac{\Delta Y}{\Delta G^*}$ above, we find that

$$\frac{\Delta Y}{\Delta C_o} = \frac{1}{1-c(1-x)}.$$

It follows directly that

$$\frac{\Delta T}{\Delta C_o} = \frac{x}{1-c(1-x)}.$$

As a general rule, it is possible to derive multipliers showing the effects on any of the variables determined by the model (the variables Y, Y_d, T, C, I, and G in this case) of a change in any of the variables which are set by outside forces (C_o, I_o, T^*, and G^* here).

A numerical example may be helpful at this point. Suppose the marginal propensity to consume out of disposable income is 75 percent ($c = .75$) while the tax system is such that taxes tend to increase by 20 percent of any rise in GNP ($x = .2$). Suppose further that $C_0 = 70$, $T^* = -40$, $I_0 = 145$, and $G^* = 155$ (amounts expressed in billions of dollars). In this case the equations (2.1) to (2.6) become:

$$
\begin{aligned}
C &= 70 + .75Y_d \\
Y_d &= Y - T \\
T &= -40 + .2Y \\
I &= 145 \\
G &= 155 \\
Y &= C + I + G
\end{aligned}
$$

The multiplier relating changes in GNP to changes in government purchases (or investment, or autonomous changes in consumption) is

$$\frac{\Delta Y}{\Delta G^*} = \frac{1}{1 - c(1-x)} = \frac{1}{1 - .75(1 - .2)} = 2.5,$$

and equilibrium income, calculated from (2.9), is

$$
\begin{aligned}
Y &= 2.5\,[400], \text{ or} \\
Y &= \$1{,}000 \text{ billion.}
\end{aligned}
$$

The values of all the variables, which can easily be calculated from the above equations, are given in the first ("original equilibrium") column of Table I. Two additional variables, not referred to earlier, are shown in the table: private saving and government deficit. Private saving is simply the difference between disposable income and consumption and amounts to \$140 billion. The government surplus (taxes minus expenditures) is \$5 billion. It may be noted that private saving (\$140 billion) plus the government surplus is equal to investment (\$145 billion). This is the equivalent of the well-known proposition that "saving must equal investment" for an economy containing a government sector.

Now suppose government purchases of goods and services increase by \$20 billion from the original level of \$155 billion per year to a new annual level of \$175 billion, and remain there. Since the multiplier for government purchases is 2.5, income will rise by \$50 billion to a new equilibrium value of \$1,050 billion. The new values of all variables are shown in the second ("new equilibrium") column of Table I, and the changes from the original position are shown in the last column.

The new equilibrium will not, of course, be reached immediately. The movement of GNP and its components to the new level involves a complex and time-consuming set of economic adjustments. The chain starts when the increased government purchases stimulate production and employment, which adds directly to GNP. Incomes are raised; a portion of the additional

TABLE I

Numerical Example of Multiplier for Government Expenditures in Model II
(amounts in billions)

	Original Equilibrium	New Equilibrium†	Change
Gross national product (Y)	\$1,000	\$1,050	+ \$50
Consumption (C)	700	730	+ 30
Investment (I)	145	145	0
Government purchases (G)	155	175	+ 20
Taxes (T)	160	170	+ 10
Disposable income (Y_d)	840	880	+ 40
Private saving ($Y_d - C$)	140	150	+ 10
Government deficit ($G - T$)	−5	5	+ 10

†After an increase of \$20 billion in the rate of government purchases.

income, 20 percent in this case, is paid over to the government in taxes; of the remaining 80 percent, 25 percent is saved, and the other 75 percent—which amounts to 60 percent (75 percent of 80 percent) of the rise in GNP—is spent on consumption, thereby stimulating further production and employment in industries producing consumer goods. The process continues through repeated "rounds" of spending and respending until GNP has been raised by \$50 billion (the multiplier of 2.5 times the initial increase of \$20 billion in government purchases). The time lags and the speed with which the adjustment to the new level of GNP can be expected to take place are discussed below.

A reduction in the level of taxation—that is, a change in T^*—would also raise GNP. In this case, if $c = .75$ and $x = .2$, we have, as indicated earlier,

$$\frac{\Delta Y}{\Delta T^*} = \frac{-c}{1 - c(1 - x)} = \frac{-.75}{1 - .75(1 - .2)} = -1.875.$$

Thus a cut in taxes of \$20 billion ($\Delta T^* = -20$) would raise GNP by \$37.5 billion. The multiplier applicable to a tax cut is smaller in absolute value (1.875) than that applicable to an increase in government purchases (2.5). The reason is that the entire increase in government purchases is a direct increase in GNP, while a portion of the tax cut is saved, and only the part that is spent on consumption (75 percent in this case) adds to GNP. It is suggested that the student work out a table similar to Table I above to illustrate the effects on income and the other variables determined within the model of a tax cut of \$20 billion.[2]

Model II illustrates in a simple way the rationale for the use of fiscal

[2]For a detailed discussion of the way in which a tax cut can increase aggregate demand and employment, see the selections entitled "The Effects of Tax Reduction on Output and Employment," by the Council of Economic Advisers, and "Measuring the Impact of the 1964 Tax Reduction," by Arthur Okun, in Chapter 4 below.

policy—changes in government expenditures and taxes—to regulate aggregate demand for goods and services in the interest of full employment and price stability. This subject is taken up in considerable detail in Chapter 4. The model used in this illustration is substantially oversimplified. In practice, for example, not all taxes are levied on households—there are direct and indirect taxes on business as well—and some saving is done by businesses as well as by households. Despite the added complexities, however, the multiplier of 2.5 for government purchases that we used above is fairly realistic. Estimates obtained by sophisticated statistical techniques applied to much more complicated models have fairly consistently turned out to be in this neighborhood.

Model III: The Introduction of Money and Interest

The student will no doubt have noticed that we have not yet mentioned money or interest rates. It is now time to remedy this deficiency in our analysis. The essence of the problem can be handled quite well and without greatly complicating the presentation by adding additional variables and equations to Model II. The resulting Model III, which takes account of money and interest, includes the following equations, five of which (the first three and the fifth and sixth) are exactly the same as those of Model II.

$$C = C_0 + cY_d \tag{3.1}$$
$$Y_d = Y - T \tag{3.2}$$
$$T = T^* + xY \tag{3.3}$$
$$I = I_0 - vr \tag{3.4}$$
$$G = G^* \tag{3.5}$$
$$Y = C + I + G \tag{3.6}$$
$$M_d = M_0 + kY - mr \tag{3.7}$$
$$M_s = M^* \tag{3.8}$$
$$M_d = M_s \tag{3.9}$$

Here, r is the interest rate (there is assumed to be only one interest rate). v is the slope of the investment function with respect to the interest rate, or, in Keynesian terminology, the slope of the marginal efficiency of capital (or investment) schedule. v is assumed to be greater than zero, but it carries a negative sign in the investment function—i.e., the lower the interest rate the more investment. M_d is the quantity (stock) of money (demand deposits and currency) demanded by the public and is assumed to be related positively to income and negatively to the interest rate. k is the number of dollars by which the public will desire to increase its money holdings per dollar increase in GNP (i.e., the slope of the money demand function with respect to income). k is, of course, positive—i.e., the higher the level of income the more money the public will want to hold (at a given interest rate). m is the slope of the demand for money function with respect to the interest rate. m is assumed greater than zero, but it carries a negative sign in the money demand function—i.e., the lower the interest rate, the more money the public will want to hold (at a given income). M_0 is the amount of money demanded without regard to income or the rate of interest; its level is determined by forces outside of the model. M_s is the supply of money; it is equal to a constant, M^*, which can be changed at will by the monetary authorities (e.g., the Federal Reserve System) through actions such as open market operations, changes in the discount rate, or changes in the reserve requirements of the banks.

Model III is changed from Model II by introducing the interest rate into the investment equation (3.4) and by introducing three new equations, (3.7), (3.8), and (3.9), to represent the "monetary sector" of the economy. Equation (3.9) is an equilibrium condition which says that the demand for money must be equal to the supply of money in order for an equilibrium to exist.

Substituting (3.2) and (3.3) into (3.1), we obtain

$$C = C_0 - cT^* + c(1-x)Y. \tag{3.10}$$

Then, substituting (3.4), (3.5), and (3.10) into (3.6), we obtain

$$Y = C_0 - cT^* + c(1-x)Y + I_0 - vr + G^*, \tag{3.11}$$

or, solving explicitly for r in terms of Y,

$$r = \frac{C_0 - cT^* + I_0 + G^*}{v} - \frac{1-c(1-x)}{v}\,Y. \tag{3.12}$$

Next, substituting (3.7) and (3.8) into (3.9), we obtain

$$M^* = M_0 + kY - mr, \tag{3.13}$$

or, solving explicitly for r in terms of Y,

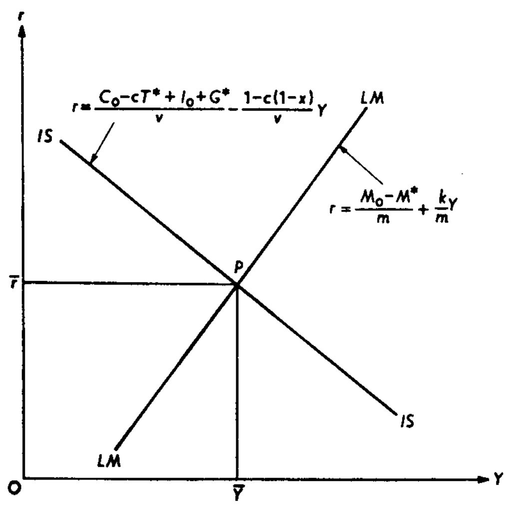
$$r = \frac{M_0 - M^*}{m} + \frac{k}{m}\,Y. \tag{3.14}$$

Equation (3.12) is the *IS* curve discussed in the Holbrook and Smith articles in this chapter. It is derived from equations (3.1) to (3.6) in the above model and represents the various combinations of income and the interest rate that will equilibrate the market for goods and services—that is, will result in aggregate demand $(C + I + G)$ being equal to total output (Y). The slope of the line $(\Delta r/\Delta Y)$ is $-[1-c(1-x)]/v$. Since c and x are both less than unity, $1-c(1-x)$ is necessarily positive, as is v. Consequently, the slope of the *IS* curve is negative—that is, it slopes downward to the right. The commonsense economic explanation is that a reduced rate of interest will lead to more investment, which, through the multiplier, will raise income; thus, a fall in the rate of interest will be associated with a higher level of income. The *IS* curve is shown as a downward-sloping line in Chart I.

Equation (3.14) is the *LM* curve, also discussed in the Holbrook and Smith articles. It is derived from equations (3.7) to (3.9) and represents the various combinations of Y and r that will result in equilibrium in the money market—i.e., equality of demand for and supply of money—with the given stock of money, M^*. The slope of the LM curve ($\Delta r / \Delta Y$) is k/m. Since k and m are both positive numbers, the slope must be positive. It is useful to think of money holdings as consisting of two parts: transactions balances required for the conduct of current economic activity by households and firms and asset balances held as a part of wealth portfolios. The demand for transactions balances may then be regarded as related positively to income, and the demand for asset balances as being related negatively to the interest rate. (The demand for money is discussed at some length in Ronald L. Teigen's paper in this chapter.[3]) Then, moving along the LM curve, a rise in income will increase the transactions demand for money, thereby leaving a smaller portion of the fixed supply (M^*) of money available to satisfy the asset demand and causing the interest rate to rise as asset holders attempt to restore portfolio equilibrium by selling bonds. The LM curve is shown as an upward-sloping curve in Chart I.

CHART I

Determination of Income and Interest Rate by IS and LM Curves



Equilibrium for the entire economy—including both the market for goods and services and the money market—occurs at the point of intersection of the IS and LM curves. This is point P in Chart I, and the equilibrium values of GNP and the interest rate are $\bar{Y}$ and $\bar{r}$.

[3] It should be noted that, as explained in Teigen's paper, some economists believe that the demand for money is almost entirely a transactions demand but that the transactions demand is dependent on both income and the interest rate. This leads to essentially the same conclusions about the functioning of money in the economy that are reached if the demand consists of a transactions component dependent on income and an asset component dependent on the interest rate.

The equilibrium level of GNP can be derived explicitly by eliminating r between equations (3.12) and (3.14). When this is done, we have

$$\frac{C_o - cT^* + I_o + G^*}{v} - \frac{1-c(1-x)}{v}Y = \frac{M_o - M^*}{m} + \frac{k}{m}Y,$$

or, solving explicitly for Y,

$$Y = \frac{1}{1-c(1-x)+\frac{vk}{m}}\left[C_o - cT^* + I_o + G^* - \frac{v}{m}(M_o - M^*)\right]. \tag{3.15}$$

This model contains three policy instruments which the authorities can adjust in order to control aggregate demand: the fiscal authorities can change government expenditures (G^*) or the tax level (T^*), while the monetary authorities can adjust the stock of money (M^*). Multipliers which show the leverage of each of these instruments in changing GNP can be calculated quite easily. For example, the multiplier for government expenditures ($\Delta Y / \Delta G^*$) can be derived as follows: suppose the level of government purchases of goods and services is increased from G^* to $G^* + \Delta G^*$. The new level of GNP is given by

$$Y + \Delta Y = \frac{1}{1-c(1-x)+\frac{vk}{m}}\left[C_o - cT^* + I_o + G^* + \Delta G^* - \frac{v}{m}(M_o - M^*)\right]. \tag{3.16}$$

Subtracting (3.15) from (3.16) and dividing through by ΔG^*, we have

$$\frac{\Delta Y}{\Delta G^*} = \frac{1}{1-c(1-x)+\frac{vk}{m}}. \tag{3.17}$$

By a similar procedure, the multipliers for changes in taxes and in the stock of money can be derived:

$$\frac{\Delta Y}{\Delta T^*} = \frac{-c}{1-c(1-x)+\frac{vk}{m}} \tag{3.18}$$

$$\frac{\Delta Y}{\Delta M^*} = \frac{1}{[1-c(1-x)]\frac{m}{v}+k} \tag{3.19}$$

Comparing the multiplier for government purchases (3.17) with that developed in Model II above (2.11), we find that the difference consists in the presence of the additional term vk/m in the denominator of (3.17). Since v, m, and k are all positive, the term vk/m is positive. It increases the denominator of (3.17) and therefore reduces the size of the multiplier. This term arises from the existence of monetary forces in Model III which were not included in Model II. In deriving expression (3.17) for the multiplier effects of a change in government purchases in Model III, it was assumed that the stock of money, M^*, was held constant. An increase in government purchases increases GNP, and the rise in GNP increases the demand for money for transactions purposes. With a constant stock of money, the needed transactions balances must be obtained from asset balances, and this necessitates a rise in the interest rate. This rise in the interest rate, in turn, reduces investment expenditure, thereby canceling out a portion of the effect of the initial increase in government purchases and cutting down the size of the multiplier.

The relationships can perhaps best be understood by means of a numerical illustration. Suppose, as in the example used to illustrate Model II, that $c = .75$, $t = .2$, $C_0 = 70$, $T^* = -40$, and $G^* = 155$. In addition, in this case let us suppose that the interest slope of the investment equation (3.4) is -4 (i.e., $v = 4$); the constant term of the investment equation, I_0, is 165; the income slope of the money demand equation (3.7), k, is .25; the interest slope of the money demand equation is -10 (i.e., $m = 10$); and the constant term of this equation, M_0, is 20. Finally, suppose the stock of money, M^*, is 220. In this case, the equations (3.1) to (3.9) become:

$$
\begin{aligned}
C &= 70 + .75Y_d \\
Y_d &= Y - T \\
T &= -40 + .2Y \\
I &= 165 - 4r \\
G &= 155 \\
Y &= C + I + G \\
M_d &= 20 + .25Y - 10r \\
M_s &= 220 \\
M_d &= M_s
\end{aligned}
$$

The multiplier for government purchases (or investment) is

$$
\frac{\Delta Y}{\Delta G^*} = \frac{1}{1 - c(1 - x) + \frac{vk}{m}} = \frac{1}{1 - .75(1 - .2) + \frac{4(.25)}{10}} = 2,
$$

and equilibrium income, calculated from (3.15), is:

$$
\begin{aligned}
Y &= 2[500] \\
Y &= \$1{,}000 \text{ billion.}
\end{aligned}
$$

The values of all the variables are given in the first column of Table II.

Now let us suppose that government expenditures increase by \$20 billion from the original rate of \$155 billion to \$175 billion. Since, as we have seen, the multiplier is 2, this will raise GNP by \$40 billion. The new values of all the variables are shown in the second column of Table II. The main

TABLE II

Numerical Example of Multiplier for Government Expenditures in Model III

(dollar amounts in billions)

	Original Equilibrium	New Equilibrium†	Change
Gross national product (Y)	$1,000	$1,040	+ $40
Consumption (C)	700	724	+ 24
Investment (I)	145	141	− 4
Government purchases (G)	155	175	+ 20
Taxes (T)	160	168	+ 8
Disposable income $(Y-T)$	840	872	+ 32
Saving (Y_d-C)	140	148	+ 8
Government deficit $(G-T)$	−5	7	+ 12
Interest rate (r)	5%	6%	+ 1%

†After an increase of $20 billion in the rate of government purchases.

difference between the results shown here and those produced by an increase of $20 billion in government expenditures in Model II (see Table I) is that in this case the rise in GNP increases the demand for money and drives up the interest rate from 5 to 6 percent, and this, in turn, reduces investment by $4 billion. That is why the multiplier is only 2 instead of 2.5 as in Model II.

Our illustration can also be presented in terms of *IS* and *LM* curves. In the original situation (before the increase in government expenditures), substitution into equation (3.12) yields the following numerical *IS* curve:

$$r = 105 - .1Y.$$

Similarly, the *LM* curve, obtained by substitution into (3.14), is

$$r = -20 + .025Y.$$

These two curves have been plotted in Chart II as the lines IS_1 and *LM*. Their intersection (point *P*) yields equilibrium values of $1,000 billion for GNP and 5 percent for the interest rate.

An increase of $20 billion in government expenditures shifts the *IS* curve to the right, and its equation becomes

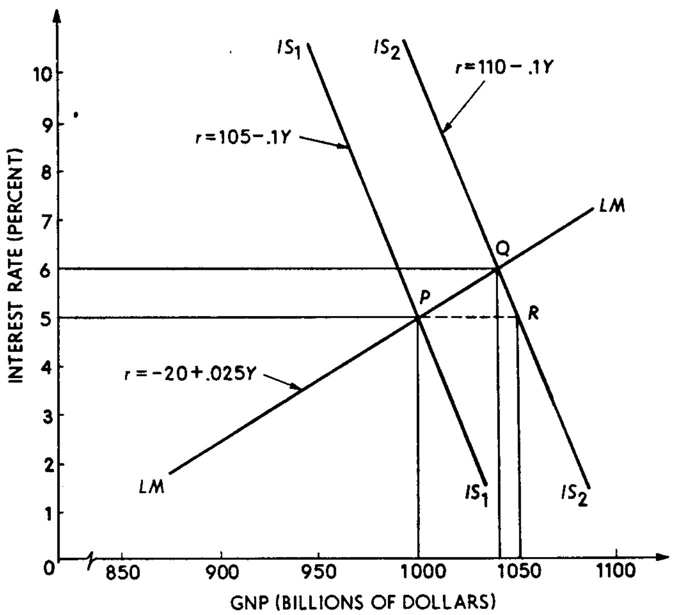
$$r = 110 - .1Y.$$

This is plotted as line IS_2 in Chart II. Its intersection with the *LM* curve (point *Q*) yields the equilibrium values of $1,040 billion for GNP and 6 percent for the interest rate.

As can be seen from Chart II, if the interest rate had not risen when government expenditures were increased, the equilibrium point would have moved from *P* to *R* and GNP would have risen by $50 billion for a multiplier of 2.5—the same as the multiplier in Model II. But due to the tightening of credit in the face of a fixed money supply, the equilibrium point moves to *Q* rather than *R*, GNP rises by $40 billion instead of $50 billion, and the interest rate rises from 5 to 6 percent. Thus, the operation of the "monetary effect" cuts the multiplier by 20 percent below what it would have been in the absence of the effect.

CHART II

Numerical Illustration of *IS* and *LM* Curves



It will be useful to consider also the effects produced by an increase in the money stock. Suppose the stock is increased by $20 billion as a result, let us say, of open market purchases of U.S. government securities by the Federal Reserve System. The multiplier applicable to an increase in the money stock, according to (3.19) above, is

$$\frac{\Delta Y}{\Delta M^*} = \frac{1}{[1 - c(1 - x)]\frac{m}{v} + k}.$$

Using the values of our illustration, this becomes:

$$\frac{\Delta Y}{\Delta M^*} = \frac{1}{[1 - .75(1 - .2)]\frac{10}{4} + .25} = \frac{1}{1.25}$$

$$\frac{\Delta Y}{\Delta M^*} = .8$$

Thus, an increase of $20 billion in the money stock will increase GNP by $16 billion. The effects on all of the variables in the system are shown in the second column of Table III. For purposes of comparison, the original values are again shown in the first column. The increase in the money stock produces its effects by lowering the interest rate from 5 percent to 3.4 percent, thereby stimulating investment; the rise in investment spending stimulates production and income, setting off a multiplier effect which raises consumption. It is interesting to note that the increase in income leads to a rise in tax collections which reduces the government deficit.

TABLE III
Numerical Example of Multiplier for an Increase in the Money Stock in Model III
(dollar amounts in billions)

	Original Equilibrium	*New Equilibrium*†	*Change*
Gross national product *(Y)*	\$1,000	\$1,016.0	+\$16.0
Consumption *(C)*	700	709.6	+ 9.6
Investment *(I)*	145	151.4	+ 6.4
Government purchases *(G)*	155	155.0	0
Taxes *(T)*	160	163.2	+ 3.2
Disposable income $(Y-T)$	840	852.8	+ 12.8
Saving (Y_d-C)	140	143.2	+ 3.2
Government deficit $(G-T)$	−5	−8.2	− 3.2
Money stock (M^*)	220	240.0	+ 20.0
Interest rate *(r)*	5%	3.4%	− 1.6%

†After an increase of \$20 billion in the money stock.

According to our analysis of Model III, it is possible to change aggregate demand and GNP by fiscal policy measures—changes in government expenditures or in taxes—or by monetary policy—changes in the monetary stock. (It is suggested that the student work out another table similar to Tables II and III summarizing the effects of a \$20 billion tax cut and compare the results with those produced by the other measures.) And, of course, the three types of measures could be combined in various ways to produce a desired effect on GNP. (Another suggested exercise for the student: calculate the size of the increase in the money stock that would be needed to accompany an increase in government expenditures in order to hold the interest rate at 5 percent and achieve a multiplier effect of 2.5.) Choice of the proper combination in given circumstances would depend on various considerations—the relative speeds with which they produce their results, their effects on goals other than the level of GNP, such as the rate of long-term growth, the nation's balance-of-payments position, and so on. Many of these considerations are discussed in readings in this book, especially in Chapters 4, 5, and 7. In particular, the introduction to Chapter 7 contains an extensive discussion of the relationship between multiple policy goals and the instruments of policy. The discussion is based on a linear model which is almost identical to Model III; thus a thorough understanding of the material in the present section will be particularly useful to the student in studying the material in Chapter 7.

Model III provides a useful starting point for a discussion of some of the major doctrinal controversies that have plagued the subject of money, particularly those having to do with the relative efficacy of monetary policy and fiscal policy. The policy controversies which we wish to examine here can conveniently be summarized as the opposing views of two schools of thought. The *Classical quantity theorists* may be said to believe that fiscal policy can have no significant and lasting effect on real output or employment, while monetary policy is viewed by members of this school of thought as being very potent in terms of its effects on these variables. At the other end of the spectrum are some of the *extreme versions of Keynesianism,* which view monetary policy as impotent and fiscal policy as being extremely effective. To see the basis for these views in terms of the analysis we have used so far, it will be useful to bring together once again the multipliers for changes in government expenditures and for changes in the money stock (assuming, as we have up to now, that the entire money stock is under the direct control of the monetary authorities—a qualification which we later will relax). These multipliers are:

$$\frac{\Delta Y}{\Delta G^*} = \frac{1}{1 - c(1-x) + \dfrac{vk}{m}} \tag{3.17}$$

$$\frac{\Delta Y}{\Delta M^*} = \frac{1}{[1 - c(1-x)]\dfrac{m}{v} + k} \tag{3.19}$$

At the beginning of this discussion, it should be emphasized that these multipliers describe very simple economies and in particular that, in these economies, prices and wages remain unchanged as income changes in response to policy or for any other reason. That is, these economies have significant amounts of unemployed resources, so that increasing demand is reflected in rising employment rather than price inflation. Such chronic underemployment is ascribable to a rigidity or inflexibility in the system—in this case, wage rates which are not free to respond to the relationship between the supply of and demand for labor. If the price of unemployed productive factors—i.e., the wage rate—were free to move as unemployed labor competed for work with those employed, presumably a wage could be found at which all of the workers who desired work could get it; however, if the wage rate is fixed by law or restricted in movement by some other arrangement, those out of work who want jobs may not be able to find them. Wage inflexibility and other rigidities which hinder the achievement of full employment undoubtedly are present in the real world, and it is therefore appropriate to conduct our analysis using a model which recognizes that they exist. The student should, however, be aware that theoretical analysis is sometimes also carried out using macroeconomic models which do not reflect these impediments to full employment; in effect, such models assert that full employment rather than underemployment is the normal state of affairs. If there exists more or less continuous full employment, it is apparent that the results of the application of monetary and fiscal policy will be different from the results in an economy which is chronically underemployed.

It is also worth observing at the outset that conclusions regarding the effectiveness of policy based on models of the simple types presented here are likely to be somewhat sensitive to the degree of detail included. Innumerable marginal details about the world are of course omitted because they would greatly increase the algebraic complexity of the analysis without adding anything of substance to the conclusions. There is, however, an important detail which has been overlooked in most discussions about policy but which has an important effect on the results of the analysis: the question of the interest sensitivity of the supply of money. Its implications for the controversy over policy which has been focused on the significance of the interest sensitivity of the *demand* for money will be examined below.

With these thoughts in mind, let us turn to the Classical view as summarized above. In one version of the Classical model which we shall consider, the treatment of the demand for money differs substantially from the approach taken in Keynesian analysis. The Classical economists generally postulated that money balances were held only to finance the transactions of households and business and that the quantity of money demanded therefore depended only on transactions or income. Since money yielded no return, the possibility that the demand for money might also depend on the interest rate was not considered by most of them. Thus, in this version of the Classical model, the interest sensitivity of the demand for money balances, m, is assumed to be zero, and (3.7) reduces to:

$$M_d = kY\,. \qquad (3.20)$$

This is the so-called "quantity theory of money" equation. A model containing a demand-for-money function with an interest sensitivity of zero is generally viewed as belonging to the Classical approach,[4] and in modified form the "quantity theory" underlies the analysis presented in the selection by Milton Friedman which is reprinted in this chapter.[5] While assuming the

[4]As we shall see below, this statement is subject to qualification. First, if prices and wages are assumed to be flexible, the Classical conclusions about the effectiveness of monetary and fiscal policy hold even if the demand for money is interest sensitive – fiscal policy cannot change real income or employment, but merely shifts the mix between government and private spending, while changes in the money stock merely result in proportional changes in money wages and prices. Second, the Keynesian conclusions hold for an underemployed economy even if the interest sensitivity of the demand for money is zero, if the interest sensitivity of the *supply* of money is not zero.

[5]The quantity theory can be expressed in two equivalent ways. The form given above, $M = kY$, is often referred to as the "Cambridge equation" because it reflected the thinking of a group of economists at Cambridge University. The other version, which is used by Friedman in the paper included in this chapter, is $MV = Py$, where V stands for the income velocity of money, P is the price level, and y is the real income ($Y = Py$). This version is often called the "Fisher equation," after the late Professor Irving Fisher of Yale. The assumption underlying both is that the demand for money is not related to the rate of interest; if it were, the behavioral assumption that velocity is constant, which is crucial to the quantity theory, is lost. Given the assumption that the demand for money is a pure transactions demand, $MV = Y$ is equivalent to $M = kY$; i.e., $k = 1/V$.

interest elasticity of the demand for money to be zero may not seem to represent a major change in the model, this difference actually is crucial in terms of assessing the relative usefulness of monetary and fiscal policy in a chronically underemployed economy in which the supply of money is under the direct control of the central bank.[6]

It will be convenient to discuss monetary policy and fiscal policy in such an economy for the extreme cases in which the interest elasticity of the demand for money is zero in value, and in which it is infinitely large. In the version of the Classical case in which m, and hence this elasticity, is zero, it is seen from (3.17) that vk/m becomes infinitely large, and the fiscal policy or expenditure multiplier $\Delta Y/\Delta G^*$ becomes zero. In such a world, therefore, fiscal policy is ineffective; for example, an increase in government spending with no change in the money stock would necessarily raise the interest rate enough to depress private investment as much as government spending had increased, thereby merely reallocating resources from the private to the public sector but having no net effect on aggregate demand, output, or employment. From (3.19), however, it is apparent that the monetary policy multiplier, $\Delta Y/\Delta M^*$, reaches its maximum possible value, $1/k$, when m is zero. (Note: In the above numerical example, the money stock multiplier becomes 4 if $m=0$—in contrast to the value of .8 that we calculated.) As m rises in value from zero, the term $[1-c(1-x)]\,m/v$ increases, reducing the value of this multiplier. The result that $\Delta Y/\Delta M^* = 1/k$, or that $\Delta M^* = k\Delta Y$, follows directly from the quantity theory equation, $M = kY$. On this basis, therefore, the typical Classical policy prescription for changing the level of money income is to use only monetary policy. While pamphleteers and popular writers had advocated government spending as a means of dealing with unemployment for many years, it is not surprising that this approach was not accepted by respectable professional economists until Keynes introduced the interest rate as a determinant of the demand for money in his *The General Theory of Employment, Interest and Money* in 1936. Nor is it surprising that modern proponents of the Classical theory, such as Friedman, attach little or no importance to fiscal policy as a means of influencing aggregate demand (although, as we shall see below, members of this school of thought—particularly Friedman—have recently attempted to argue that fiscal policy is impotent while recognizing that the demand for money is interest sensitive).

The conclusion that aggregate income changes in direct proportion to the money stock if m is zero (assuming also, as above, that the supply of money

[6]The coefficient m represents the change in the demand for money as an asset corresponding to a unit change in the interest rate; that is, $\Delta M = -m\Delta r$, or $\Delta M/\Delta r = -m$. The *elasticity* of demand for money with respect to the interest rate ($\eta_{M^D,r}$) is the *percentage* change in the demand for money divided by the *percentage* change in the interest rate. Thus:

$$\eta_{M^D,r} = \frac{\Delta M/M}{\Delta r/r} = \frac{\Delta M/\Delta r}{M/r} = -\frac{m}{M/r}.$$

Since m appears in the numerator of this expression, when m takes the extreme values of zero or infinity, the elasticity also is zero or infinity.

does not respond to interest rate changes) is based on the usual comparative static equilibrium analysis, and does not take the dynamics of the system into account. Friedman agrees with the spirit of the Classical approach, but goes further and argues that monetary policy is not only powerful but erratic: while changes in the money stock have a strong leverage on income, the behavioral lags are variable and undependable so that, in some cases, the effect on income may be rapid, while in others it may be very slow.[7] This leads to his recommendations that discretionary monetary policy be abandoned and a "rule" providing for a constant percentage growth in the money stock be substituted for it, a proposal which has been debated with increasing intensity recently.[8] Friedman's views concerning lags are hotly disputed by many monetary economists.

In other writings, Friedman has contended that his views regarding monetary and fiscal policy do not depend critically on the absence of a significant interest elasticity of demand for money. Indeed, he has expressed the view that the demand for money should, in principle, be responsive to interest rate changes. In his own empirical work, he does not find evidence in support of a significant degree of interest sensitivity.[9] But, in this respect, his results differ from those of most other investigators, as he has recognized.[10] Some of the evidence from studies which find significant interest sensitivity is summarized in Teigen's paper in this chapter. We have seen that if rigidity of money wage rates creates impediments to the automatic achievement of full employment, *both* monetary and fiscal policy will be capable of changing real income and employment unless the interest elasticity of demand for money is zero. However, as Friedman has pointed out, the situation is different if money wages are flexible.[11] Accordingly, it is useful to examine the effects of monetary and fiscal policy under these conditions.

As is shown in the papers by Holbrook and Smith in this chapter, if money wages are flexible—that is, if they decline readily when the number of persons willing to work at the going wage rate exceeds the number of jobs available—the economy will normally tend automatically toward full employment regardless of the monetary and fiscal policies being followed. That is, real output and employment will be determined by the volume of real resources available and their productivity. The Classical quantity theory of money will hold in its extreme form: an increase in the stock of money will cause an equal proportional change in prices and in money income but will leave real income and employment unchanged. This will be true without

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[7]The empirical evidence on which this conclusion is based is presented in Milton Friedman, "The Supply of Money and Changes in Prices and Growth," in *The Relationship of Prices to Economic Stability and Growth,* Compendium of Papers Submitted by Panelists Appearing before the Joint Economic Committee (Washington, D.C., 1958), pp. 249-50.

[8]Chapter 5 contains readings which are concerned with this debate.

[9]See Milton Friedman, "The Demand for Money: Some Theoretical and Empirical Results," *Journal of Political Economy,* Vol. LXVII (August 1959), pp. 327-51. In order to get this result, Friedman uses a special definition of income.

[10]See Milton Friedman, "Interest Rates and the Demand for Money," *Journal of Law and Economics,* Vol. IX (October 1966), pp. 71-85.

[11]*Ibid.*

regard to the magnitude of the interest elasticity of demand for money.[12] Fiscal policy will likewise not affect real income or employment. If government expenditures are increased without a corresponding increase in taxes, the government will have to borrow in the capital market to finance the resulting deficit. The additional government borrowing will necessarily raise interest rates enough to cause real private investment to decline as much as real government expenditures increase. The result will be a transfer of real resources from the private to the public sector of the economy, but total income and employment will be unaffected. Thus, as Friedman points out, fiscal policy will be incapable of affecting income and employment under a regime of flexible wages, whether or not the demand for money is sensitive to interest rates.

It should be noted, however, that even if wages are flexible, the effects of fiscal policy do depend to some degree on whether the demand for money is sensitive to interest rates. If the demand for real money balances depends only on real income and not on interest rates, the price level and therefore the level of money income is determined by the stock of money alone. In this case, a change in fiscal policy will leave the price level and money income unchanged. If, however, the demand for money is sensitive to interest rates, the outcome will be different. An increase in government expenditures not accompanied by an increase in taxes will, as explained earlier, cause interest rates to rise. If the demand for money is sensitive to interest rates, the rise in interest rates will cause a reduction in the amount of real money balances people want to hold (remember that real income is unchanged because it is determined by the amounts of real resources available and their productivity). With the nominal stock of money unchanged (by assumption), the price level will have to rise enough to bring the real value of cash balances into alignment with the reduced demand for such balances.[13] Thus, fiscal policy will, in this case, affect the price level and money income. To summarize: If wages are flexible, fiscal policy will be incapable of affecting real income and employment, whether or not the

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[12]Provided the interest elasticity is not infinitely large—if this is the case, as Smith's article shows, the Classical mechanism which makes the economy tend automatically toward full employment may break down. (In this discussion, we are neglecting the so-called "Pigou effect," which is discussed in Smith's article.)

[13]This can be seen by examining the *LM* curve for a model similar to those presented above except that prices are now assumed to be free to change. The equation of the *LM* curve is derived by equating the demand for real money balances (which is assumed to depend on real income (Y/P) and the rate of interest) with the real value of the nominal money stock (M^*/P). Thus we have

$$\frac{M^*}{P} = M_o + k\frac{Y}{P} - mr.$$

If real income (Y/P) is fixed at the full-employment level and M^* is given, an increase in government expenditures which causes the interest rate to rise will reduce the demand for real money balances (the right-hand side of the equation). In order to maintain equilibrium in the money market, the supply of real money balances will have to be reduced also, and with M^* fixed this reduction can only be brought about by a rise in P. If P rises, Y must rise in the same proportion if Y/P is to remain constant. Thus, fiscal policy affects money income and prices. However, if $m = 0$ so that the interest rate does not affect the demand for money, none of these adjustments is necessary, and fiscal policy leaves money income and prices unaffected.

demand for money is elastic to interest rates; however, even with flexible wages, fiscal policy will have effects on the price level and money income unless the interest elasticity of demand for money is zero.

Assuming that the demand for money is sensitive to interest rates, as the bulk of the empirical evidence indicates, the above discussion raises an important question. Which of the following alternative assumptions is the more realistic: *(a)* wages are fully flexible so that monetary and fiscal policy are incapable of changing employment and real income, having effects only on the price level and money income; or *(b)* wages are rigid (or at least sticky), thereby giving monetary and fiscal policy an important leverage over employment and real income? It seems clear that in a world of imperfect markets, less than full mobility of resources, trade unions, minimum-wage laws, and the like, the rigid-wage assumption is by far the more reasonable one to adopt. Accordingly, we shall continue to assume rigid wages during the remainder of this introduction, deferring further discussion of wages and prices until the second part of this chapter.

There is still another dimension to the controversy over the role of interest rates in relation to monetary and fiscal policy—a dimension which has received less attention than it deserves. We refer here to the responsiveness of the money *supply* to changes in the interest rate through the operation of the banking system. Up to this point in our discussion, for simplicity, we have treated the stock of money, M^*, as a variable that is under the direct control of the monetary authorities. Strictly speaking, this is not correct. In the United States, the Federal Reserve System implements monetary policy primarily by buying and selling U.S. government securities in the open market, by changing the reserve requirements of member commercial banks, and by varying the discount rate at which member banks may borrow from the System. Thus, it is these variables rather than the money stock itself that are properly regarded as being under the control of the authorities. Since the amount of reserves obtained by borrowing from the Federal Reserve as well as the amount of reserves held in excess of legal requirements seem to be affected by interest rates available in the market relative to the discount rate charged by the Federal Reserve, the money supply is determined jointly by the actions taken by the authorities and the responses of the banks and the public. One of the results is that the supply of money as well as the demand for money is sensitive to interest rates. The implications for monetary and fiscal policy of this more sophisticated approach to the supply of money are discussed thoroughly in the paper by Teigen. We can bring our introductory discussion to completion by examining the results of substituting a very simple money supply equation for equation (3.8). Let us suppose that the monetary authorities are able to vary only the reserve base (R^*) and that the commercial banks extend more loans and hence increase the amount of demand deposits in response to increases in the interest rate, and vice versa. Instead of (3.8), Model III now contains the following money supply equation:

$$M_s = aR^* + er, \qquad a > 0,\ e > 0. \tag{3.21}$$

We may now derive multiplier expressions summarizing the effects of fiscal policy and monetary policy in exactly the same way as before. Instead of the expressions given by (3.17) and (3.19), we now obtain:

$$\frac{\Delta Y}{\Delta G^*} = \frac{1}{1 - c(1-x) + \frac{vk}{m+e}} \tag{3.22}$$

$$\frac{\Delta Y}{\Delta R^*} = \frac{a}{[1 - c(1-x)]\frac{m+e}{v} + k} \tag{3.23}$$

In both of these expressions, we note that the interest sensitivity of the *supply* of money (e) is combined additively with the interest sensitivity of the demand for money (m). Thus even if the latter were zero, as most of the Classical economists have assumed, fiscal policy remains effective as long as the interest sensitivity of the supply of money is not zero. In the same way, the larger is the interest elasticity of the money supply, the less potent is monetary policy, given the value of the interest sensitivity of demand. It appears, then, that the crucial question relating to policy for an economy in which there are impediments to full employment is not whether the *demand* for money is interest sensitive, but whether either the demand *or* supply of money exhibit interest sensitivity. Only if *both* sensitivities are zero do the extreme Classical conclusions hold.

The Classical position represents one extreme view of the size of the interest elasticity of the demand for money (as we have noted, both the Classical and the Keynesian schools have disregarded the interest elasticity of the money supply). The other polar view is that the interest elasticity of demand is infinitely large; this is the "extreme Keynesian" assumption, and is sometimes referred to as the "liquidity trap" case. If m were infinitely large, the expenditure multiplier (3.17) becomes $1/[1-c(1-x)]$. The monetary sector has no inhibiting effect on the expenditure multiplier at all, and fiscal policy reaches maximum effectiveness. However, the monetary policy multiplier (3.19) becomes zero. Monetary policy is completely ineffective because changes in the money stock have no effect on the interest rate and hence on expenditures. The "liquidity trap" means that increases in the money stock are simply absorbed into idle balances, since it is universally expected that the interest rate will rise and bond prices fall. If the possibility of interest-induced changes in the money supply is recognized, a glance at equations (3.22) and (3.23) indicates that an infinitely large interest elasticity of supply will yield similar results—if e is infinitely large, (3.22) becomes $1/[1-c(1-x)]$, its largest possible value, while the multiplier summarizing the effects on income of changes in bank reserves, (3.23), becomes zero. Such a result might occur in a period of deep depression when interest rates were very low and seemed almost certain to rise in the near future. Under these conditions the banks, fearing a fall in security prices, might be very reluctant to buy securities, and they might be extremely fearful of making additional loans at the prevailing very low interest rates in view of the high risks of default. Under these conditions, any increase in bank reserves, resulting, let us say, from open market operations, might merely cause the banks to add a corresponding amount to their excess reserves without leading to any increase in the money supply.

It is almost certain that neither the Classical nor the extreme Keynesian assumptions accurately characterize our economy under normal conditions. The evidence appears overwhelming that both the demand for and the supply of money possess some degree of interest elasticity but that neither of these elasticities is infinitely great.[14] With one further qualification in the case of monetary policy—that the sensitivity of investment to the interest rate is not zero—it may be seen from (3.22) and (3.23) that in this case both fiscal policy and monetary policy are capable of influencing income. Most recent studies have verified that investment (especially in new houses and in plant and equipment) responds to changes in the rate of interest. Some evidence on the interest elasticities of different categories of investment demand is presented in the article by Michael J. Hamburger in Chapter 5. In a system with nonzero but finite interest elasticities of demand for and/or supply of money, and a nonzero interest elasticity of demand for investment, the efficacy of monetary policy, as well as fiscal policy, depends on the structure of *both the real sector (i.e., the markets for goods and services) and the monetary sector.* In fact, both the expenditure multiplier and the monetary policy multiplier expressions contain the same terms: the marginal propensities to consume and to add to money balances with respect to income; the responsiveness of investment, the demand for money, and the supply of money to the rate of interest; and the marginal response of tax payments to income.[15] From (3.22) and (3.23) it is easy to determine the conditions that are conducive to the effectiveness of monetary policy and of fiscal policy, respectively. It should not be concluded from the discussion of the polar cases above that all of the conditions which are favorable to fiscal policy are unfavorable to monetary policy, or vice versa. In fact, there are a number of conditions that are conducive to the effectiveness of both kinds of policy. Both multipliers will be larger:

1. The larger the marginal propensity to consume with respect to disposable income (c), and the lower the marginal response of tax payments to income (x). This is true because these coefficients determine the size of induced expenditure and income changes set off by an initial change in government expenditures or tax collections produced by fiscal policy, or by an initial change in private investment resulting from a change in interest rates produced by monetary policy.

[14] It should be noted that Keynes himself felt that this would be the normal situation and that both fiscal and monetary policy would therefore be capable of influencing aggregate demand. He regarded the "liquidity trap" situation as one that might occur only in times of deep depression, such as the 1930's. It is totally wrong to regard the liquidity trap case as the essence of Keynes' analysis.

[15] In addition, the monetary policy multiplier, (3.23), has in its numerator the sensitivity of the money supply to changes in the reserve base (a). If the money supply does not respond to changes in the reserve base, then monetary policy as expressed through open market operations can have no effect on the interest rate, investment, and income. As Teigen's article shows, the structure of the monetary system is such that this sensitivity is not zero. As a matter of fact the coefficient a turns out to be the standard credit expansion multiplier, as Teigen demonstrates.

2. The smaller is the responsiveness of the demand for money to changes in income (k). For a given increase in income, for instance, the interest rate will rise less and induced investment spending will fall less, the less cash is drawn into transactions balances to accommodate the rising level of income.

The two multipliers are affected differently by variations in the interest sensitivity of investment and of the demand for and supply of money (v, m, and e). Fiscal policy is more effective, and monetary policy less effective:

1. The lower the interest sensitivity of investment expenditure (v). That is, the lower this sensitivity, the less will such expenditure be reduced by rising interest rates which accompany rising expenditure and income brought about by an increase in government expenditures or a reduction in taxes. On the other hand, the lower this sensitivity, the less effect will a given change in the money stock and the resulting interest rate change have on investment spending.

2. The greater the interest sensitivities of the demand for money (m) and the supply of money (e). The greater these sensitivities, the less will expenditure be reduced by rising interest rates accompanying rising expenditure and income caused by expansionary fiscal measures; but, on the other hand, the less effect will a given monetary change have on the interest rate and hence on spending.

In summary, monetary policy is more effective when the interest sensitivities of the demand for and supply of money are low, so that changes in the money stock or reserve base have greater effects on the rate of interest, and when the interest sensitivity of investment expenditure is large, so that changes in the interest rate have larger effects on spending. Fiscal policy is more effective when the interest sensitivity of the demand for and supply of money is large, so that an increase in transactions requirements for money and the resulting reduction in money balances left to satisfy asset requirements does not affect the interest rate very much and also, to some extent, induces the banks to create new money balances, and when the interest sensitivity of investment expenditure is low, so that rising interest rates do not deter very much investment expenditure.

The conclusions of this discussion of the factors influencing the effectiveness of monetary and fiscal policy—as measured by the size of the multipliers $\Delta Y / \Delta G^*$ and $\Delta Y / \Delta R^*$—are summarized in the following tabulation:

Increase in:	Effect on: $\frac{\Delta Y}{\Delta R^*} = \frac{a}{[1 - c(1-x)]\frac{m+e}{v} + k}$	Effect on: $\frac{\Delta Y}{\Delta G^*} = \frac{1}{1 - c(1-x) + \frac{vk}{m+e}}$
c	increase	increase
x	decrease	decrease
m	decrease	increase
e	decrease	increase
v	increase	decrease
k	decrease	decrease

The student will benefit, as he studies this book, from trying to see how the materials fit into the framework developed above. As has already been noted, for example, some of the material in Chapter 5 is concerned with the size of v, the interest sensitivity of spending decisions, and the discussion of the working of banks and financial intermediaries in Chapter 2 and of the "slippages" in the financial system in Chapter 5 are significant primarily because they bear on the size of m and e, the interest sensitivities of the demand for and supply of money.

2. SOME RUDIMENTARY DYNAMICS

(a) The Dynamics of the Multiplier

Changes in aggregate demand, whether produced by autonomous shifts in private spending or by changes in fiscal and monetary policy, are not reflected immediately in changes in production and income. There are three major lags in the process of income generation: (1) the lag between the receipt of income and the expenditure of that portion of it that the recipient decides to spend; (2) the lag between changes in expenditure and related changes in production and income; (3) the lag between the earning of income and its receipt. These lags have been called the *expenditure lag*, the *output lag*, and the *earnings lag*, respectively. While the lags are essentially additive, it appears that the expenditure and earnings lags are relatively short and that the output lag is by far the longest and most important.[16] It is this lag that receives most of the attention in the article by Gardner Ackley on the multiplier time period reprinted in this chapter. It arises because of the relationships between sales, inventories, and output. For example, an increase in sales of consumer goods will commonly lead to a reduction in retail inventories in the first instance. After a delay, which will depend on the practices of the industry, on marketing channels, and on the general business situation, retailers will increase their orders to restore depleted inventories, and these increased orders will in due course cause manufacturers to increase production and employment. Thus, significant changes in output usually occur only after a delay, which may be considerable.

Elementary expositions of the dynamics of income change commonly employ an expenditure lag, assuming, for example, that consumption adjusts to income with a lag of one period. However, in view of the fact that the output lag appears to be the most important and is the one stressed by Ackley, we shall build our exposition around that lag. This seems more realistic, although the algebraic results are very similar with an expenditure lag.

Model IIA: A Dynamic Version of Model II

To begin our discussion of the dynamic multiplier—that is, the process through which the system adjusts from one equilibrium position to another

[16] Lloyd A. Metzler, "Three Lags in the Circular Flow of Income," in *Income, Employment, and Public Policy: Essays in Honor of Alvin H. Hansen* (New York: W. W. Norton & Co., Inc., 1948), pp. 11-32.

when some component of spending changes—we will use a version of Model II, modified by dating all of the variables in such a way that this period's output (and hence current income) depends only on spending during the previous period. After we have become familiar with the adjustment process using this simple model, we shall turn to a version of Model III, in which the monetary sector plays a role. The dynamic version of Model II is as follows:

$$C_t = C_o + cY_{d_t} \tag{2a.1}$$

$$Y_{d_t} = Y_t - T_t \tag{2a.2}$$

$$T_t = T^* + xY_t \tag{2a.3}$$

$$I_t = I_o \tag{2a.4}$$

$$G_t = G^* \tag{2a.5}$$

$$Y_t = C_{t-1} + I_{t-1} + G_{t-1} \tag{2a.6}$$

Here the subscript t designates the value of the indicated variable in the current period, the subscript $t-1$ refers to the previous period, etc. In equilibrium, the value of each variable is unchanged from period to period. Accordingly, all time subscripts can be dropped, and the solution of this model is the same as the solution of Model II. However, the interpretation of equation (2a.6) is now different—it is no longer an "equilibrium condition" (although equilibrium values of the variables can be obtained by assuming that equilibrium exists [that is, that $C_t = C_{t-1}$; $I_t = I_{t-1}$; and $G_t = G_{t-1}$], and substituting these values into the equation). Rather, (2a.6) is a statement of the rule that firms are assumed to follow in deciding how much to produce. It says that they produce in a particular period an amount equal to their sales in the previous period; that is, output is adjusted to sales with a lag of one period. Thus, equation (2a.6) implies that an increase in spending in the current period has no effect on current production and national income; rather, inventories are drawn down to fill the new orders, and production responds during the following period. Other rules could have been specified, of course, but this one is simple and at the same time realistic enough to be useful.

In Model IIA, the length of each "period" need not correspond to any particular unit of calendar time, such as a month or year, but is determined by the behavioral lag between a change in spending and the change in production which it induces. In reality, it is a "distributed lag"; that is, the change in production does not occur entirely in one period but is spread out over a number of periods. For example, the production adjustment may begin slowly, rise to a peak, and then gradually taper off. Here we treat the production response to a given change in sales as occurring in one discrete unit time period. In tracing the path followed by income in response to a spending change, we will also assume that the marginal propensity to consume out of disposable income (c) and the marginal propensity to pay taxes out of GNP (x) do not change in value over time.

We will consider the effects of two types of spending changes, using changes in government purchases of goods and services as an example—although, of course, similar effects would be produced by shifts in consumer spending, investment spending, or tax collections. In Case 1, we will suppose that government purchases rise from G^* to $G^* + \Delta G^*$ *during one period only,* and then revert to the original level, G^*, and remain there. In Case 2, we start from the same equilibrium and suppose that government purchases rise to $G^* + \Delta G^*$ and *remain at the new level indefinitely.* In each case, we will trace the path of income from the original equilibrium level to the new equilibrium position. While we will consider the effects of an increase in spending, the same analysis will apply in reverse for a spending decrease.

CASE 1: ONE-SHOT INJECTION. Starting from an equilibrium position, suppose that, in the first period under consideration, government purchases of goods and services rise from the original level, G^*, to a new level, $G^* + \Delta G^*$—that is, government purchases change by ΔG^*. In the second period, government purchases revert to their former level, G^*, and remain there in all future periods. Because production is determined only by the previous period's spending, income—that is, the total value of current production—does not change in period 1; rather, inventories are drawn down so that the investment actually realized by firms for the period is $I_0 - \Delta G^*$, not I_0 as planned (that is, inventory investment falls by the same amount that spending by government increases). Since income does not change, there is no change in tax collections, disposable income, or consumption expenditure.

In period 2, firms continue to produce the (as yet unchanged) amounts of output corresponding to spending by households and by firms themselves for investment purposes, and also produce the total amount bought by government in period 1, $G^* + \Delta G^*$.[17] Therefore income rises by ΔG^* in period 2, and corresponding to this income increase, current tax collections rise by $x \Delta G^*$. Disposable income in period 2 changes by the amount of the change in income, ΔG^*, less the change in tax collections, $x \Delta G^*$, or by $(1-x) \Delta G^*$, and so consumption spending in the amount of $c(1-x) \Delta G^*$ is induced in period 2, drawing down inventories by this amount.[18] In period 3, firms produce an amount equal to their sales in period 2, which exceed the level prevailing in the initial equilibrium by $c(1-x) \Delta G^*$. Thus income in period 3 is greater than its initial equilibrium level by this amount; however, it is lower than income in period 2, due to an increase in household savings of $(1-c)\ (1-x) \Delta G^*$ in period 2. The increment of income $c(1-x) \Delta G^*$ in period 3 induces consumption spending of $c(1-x)[c(1-x) \Delta G^*]$, or $c^2 (1-x)^2 \Delta G^*$, etc. In general, the initial increase in spending produces an increment of induced production (and hence income) in each succeeding

[17]Note that in this model, production in each period is equal to *planned* spending by all sectors in the previous period. Thus the inventory change that occurred in period 1 is disregarded in period 2.

[18]Since firms are producing the incremental amount bought by government in period 1, there is actually a net inventory accumulation of $\Delta G^* - c(1-x) \Delta G^*$, or $[1 - c(1-x)] \Delta G^*$, during the period.

period, beginning with the period following the spending change; each increment is smaller than the one preceding it, since a part of each is drained off by the government in taxes and another part is saved by households, until finally these increments approach zero in value and equilibrium is restored *at the original level of income.*

It may help to illustrate the nature of this process if an example is employed, using the same numerical assumptions as in Model II. That is, suppose we have

$$
\begin{aligned}
C_t &= 70 + .75Y_{d_t} \\
Y_{d_t} &= Y_t - T_t \\
T_t &= -40 + .2Y_t \\
I_t &= 145 \\
G_t &= 155 \\
Y_t &= C_{t-1} + I_{t-1} + G_{t-1}
\end{aligned}
$$

The equilibrium values of the variables in this model are shown in the column labeled "original equilibrium" of Table IV below. In period 1, government purchases of goods and services rise by \$20 billion to \$175 billion from the initial level of \$155 billion; they revert to \$155 billion in period 2 and remain there in all succeeding periods. The paths followed by income and other variables are shown in columns numbered 1, 2, 3, and 4, where the column numbers correspond to the time periods. As can be seen, income rises to a peak of \$1,020 billion during period 2, and then declines until it ultimately reaches the former equilibrium level of \$1,000 billion as shown in the final ("new equilibrium") column. In the row labeled ΔY_t, the change in income in each period measured from the original equilibrium level is shown. The general expression for ΔY_t is shown in the following tabulation:

	Time period (t)					*New Equilibrium*
	1	*2*	*3*	*4*	*n*	
ΔY_t	0	ΔG^*	$c(1-x)\Delta G^*$	$[c(1-x)]^2\Delta G^* \ldots$	$[c(1-x)]^{n-2}\Delta G^* \ldots$	0

Since c and x are positive fractions between zero and unity in value, $c\,(1-x)$ is also such a fraction, and $[c(1-x)]^m$ declines steadily in value as m increases. Since the new equilibrium position is not reached until an infinite amount of time has passed, the term $[c(1-x)]^m$ goes to zero, and the new equilibrium is therefore the same as the original equilibrium that existed before the "one shot" increase in government purchases. The explanation is that, since government spending fell back to its original level in the second period of our example, and remained there, the equilibrium solution of the system is unchanged. However, the one-period spurt in spending initiated a dynamic process of adjustment which takes an infinite number of periods to complete. In each of these periods, income will be greater than equilibrium income, although it will be approaching the equilibrium value as time passes.

TABLE IV

The Dynamics of a "One-Shot" Spending Injection

	Original Equilibrium	Time Period (t) 1	2	3	4		New Equilibrium
Y	1,000	1,000	1,020	1,012.0	1,007.20	...	1,000
C	700	700	712	707.2	704.32	...	700
I	145	145	145	145.0	145.00	...	145
G	155	175	155	155.0	155.00	...	155
T	160	160	164	162.4	161.44	...	160
Y_d	840	840	856	849.6	845.76	...	840
S	140	140	144	142.4	141.44	...	140
ΔY_t†	...	0	20	12.0	7.20	...	0

† ΔY_t refers to the difference between income in the tth period and the original equilibrium value.

CASE 2: CONTINUING INJECTION. Now let us consider the dynamics of the case in which government purchases change and remain at the new level; that is, the case in which there is a change in the equilibrium level of government purchases. Let this change be represented by ΔG^* as previously (remember that shifts in consumption or private investment will have the same effect). As before, income will change by ΔG^* in period 2 as producers respond to the change in government spending in period 1. During period 2, there will be another injection of government spending greater than the old equilibrium by ΔG^*; at the same time, households will be induced to spend $c(1-x)\Delta G^*$, which in turn will induce that much production in period 3, etc. In each period after the second, income will differ from the initial equilibrium level by the sum of a new component of production for the government sector equal to the previous period's increased spending (compared to the former equilibrium level of government spending) and one or more components which correspond to induced spending by the household sector in the previous period. The income changes for several such periods are written out in Table V using the numerical version of Model IIA.

TABLE V

The Dynamics of a Continuing Spending Injection

	Original Equilibrium	Time Period (t) 1	2	3	4		New Equilibrium
Y	1,000	1,000	1,020	1,032.0	1,039.20	...	1,050
C	700	700	712	719.2	723.52	...	730
I	145	145	145	145.0	145.00	...	145
G	155	175	175	175.0	175.00	...	175
T	160	160	164	166.4	167.84	...	170
Y_d	840	840	856	865.6	871.36	...	880
S	140	140	144	146.4	147.84	...	150
ΔY†	...	0	20	32.0	39.20	...	50
$\frac{\Delta Y}{\Delta G^*}$	...	0	1	1.6	1.96	...	2.50

† ΔY refers to the difference between income in the tth period and the original equilibrium value.

To find the amount by which income has changed at the end of any period as a result of the change of government spending, all of the increments of induced production (and hence income) in that period which arise from this spending change must be added to the production which corresponds to the previous period's "new" additional government spending. For example, in period 3, income is $32 billion higher than its initial equilibrium level. This is due to the fact that, in period 2, government spending on goods and services was $20 billion higher than its initial level, inducing that much new production for the government sector in period 3; in addition, $12 billion of goods was produced in period 3 in response to the induced rise in consumption spending of $12 billion in period 2.

Model IIIA: A Dynamic Version of Model III

Now let us examine the effects on the process of income change of a shift in spending in a model containing a monetary sector. For this purpose we shall use a dynamic version of Model III. Since the adjustment of a household's or firm's cash balance is a simple matter and presumably takes less time to accomplish than either changes in spending plans or production decisions, we shall assume that there are no lags in the monetary sector. As was the case in Model IIA, therefore, the only lag in the system will be an output lag. The dynamic version of Model IIIA is as follows:

$$C_t = C_o + cY_{d_t} \tag{3a.1}$$
$$Y_{d_t} = Y_t - T_t \tag{3a.2}$$
$$T_t = T^* + xY_t \tag{3a.3}$$
$$I_t = I_o - vr_t \tag{3a.4}$$
$$G_t = G^* \tag{3a.5}$$
$$Y_t = C_{t-1} + I_{t-1} + G_{t-1} \tag{3a.6}$$
$$M_{d_t} = M_o + kY_t - mr_t \tag{3a.7}$$
$$M_{s_t} = M^* \tag{3a.8}$$
$$M_{d_t} = M_{s_t} \tag{3a.9}$$

In this discussion, we will consider only the "continuing injection" version of the process of income change, as it is the typical multiplier case. Starting from equilibrium, let there be an increase in government spending of ΔG^* in period 1. According to Model IIIA, in which income in any period responds only to spending in the previous period, this spending change will have no effect on income until period 2. In that period, producers will increase their output, and therefore income, by ΔG^*. As a consequence, changes in spending by both households and firms are induced in this period. The change in household spending can easily be deduced from the first three equations of our model. An income increase of ΔG^* causes an increase in disposable income of $(1-x)\Delta G^*$, leading to a rise in consumption spending of $c(1-x)\Delta G^*$ in the same period. To isolate the effects of a change in income on the spending decisions of firms, the investment equation (3a.4) must be examined together with the equations describing the monetary sector—(3a.7), (3a.8), and (3a.9). First, by differencing equation (3a.4), we note that

$$\Delta I_t = -v\Delta r_t, \tag{3a.10}$$

or that rising interest rates cause business firms to reduce investment, and vice versa. The effects of rising income on the rate of interest can be found from the monetary equations. Combining (3a.7), (3a.8), and (3a.9) produces the following *LM* equation:

$$M^* = M_0 + kY_t - mr_t. \tag{3a.11}$$

If we write this equation in differenced form, remembering that the money supply, M^*, is assumed to remain constant, we get:

$$0 = k\Delta Y_t - m\Delta r_t; \tag{3a.12}$$

or, solving this equation for Δr_t in terms of ΔY_t, we find

$$\Delta r_t = \frac{k}{m}\Delta Y_t. \tag{3a.13}$$

Substituting this result into (3a.10) yields

$$\Delta I_t = -\frac{vk}{m}\Delta Y_t, \tag{3a.14}$$

the general expression for the change in planned investment spending which is induced by a change in income, ΔY_t. Since the income change in period 2 is ΔG^*, the induced change in investment spending in that period is

$$-\frac{vk}{m}\Delta G^*.$$

Thus the *total* change in induced spending in period t will be

$$\left[c(1-x)-\frac{vk}{m}\right]\Delta Y_t,$$

and in period 2 it will be

$$\left[c(1-x)-\frac{vk}{m}\right]\Delta G^*.$$

In addition, the autonomous spending increase of ΔG^* is assumed to continue.

Thus the total income change in period 3, which is the sum of induced and autonomous spending changes in period 2 in our model, is

$$\left[c(1-x)-\frac{vk}{m}\right]\Delta G^* + \Delta G^*.$$

This in turn will induce further new spending in period 3, according to the rule given above, of

$$\left[c(1-x)-\frac{vk}{m}\right]\left[c(1-x)-\frac{vk}{m}\right]\Delta G^*,$$

or

$$\left[c(1-x)-\frac{vk}{m}\right]^2\Delta G^*,$$

which, when added to the autonomous spending component ΔG^* will generate further income change in period 4, etc.

It may be useful to illustrate this process with a numerical example, using a model which yields the same equilibrium values for income and their other common endogenous variables as the numerical version of Model IIA, but which contains a monetary sector whose parameters have the same values as those of Model III, as follows:

$$\begin{aligned}
C_t &= 70 + .75Y_{d_t}\\
Y_{d_t} &= Y_t - T_t\\
T_t &= -40 + .2Y_t\\
I_t &= 165 - 4r_t\\
G_t &= 155\\
Y_t &= C_{t-1} + I_{t-1} + G_{t-1}\\
M_{d_t} &= 20 + .25Y_t - 10r_t\\
M_{s_t} &= 220\\
M_{d_t} &= M_{s_t}
\end{aligned}$$

Changes in income and several other variables which result from a continuing injection of \$20 billion of new government spending initiated in period 1 are written out in Table VI.

Again we find the total amount of income change for any period due to a

TABLE VI
The Dynamics of a Continuing Spending Injection in a Model with a Monetary Sector

	Original Equilibrium	Time Period (t) 1	2	3	4		New Equilibrium
Y	1,000	1,000	1,020	1,030	1,035.0	...	1,040
C	700	700	712	718	721.0	...	724
I	145	145	143	142	141.5	...	141
G	155	175	175	175	175.0	...	175
T	160	160	164	166	167.0	...	168
Y_d	840	840	856	864	868.0	...	872
S	140	140	144	146	147.0	...	148
M	220	220	220	220	220.0	...	220
r	5%	5%	5.5%	5.75%	5.875%	...	6%
ΔY_t†	...	0	20	30	35.0	...	40
$\frac{\Delta Y_t}{\Delta G_t^*}$	...	0	1	1.5	1.75	...	2.0

† ΔY_t refers to the difference between income in the tth period and the original equilibrium value.

change in government spending by adding all of the increments of induced production (or income) in that period to the production corresponding to the previous period's "new" government spending. In the present example, for instance, income in period 3 is $30 billion higher than its original equilibrium level. This change is due to three causes: First, government spending on goods and services in period 2 was $20 billion higher than its initial level, inducing that much new production for the government sector in period 3; second, $12 billion of goods was produced in period 3 in response to the induced rise in consumption spending of $12 billion in period 2; finally, $2 billion *less* of investment goods was produced in period 3 than initially because spending by firms declined by that amount in period 2 due to a rise in the rate of interest from 5 percent to 5.5 percent. It is possible to calculate multipliers for each successive time period. This is done by dividing the rise in income for that period by the increase in the level of government purchases that caused the rise. These multipliers are shown in the $\Delta Y_t / \Delta G^*$ rows of Tables V and VI: thus, in Table VI, the multiplier is 1.00 after two periods, 1.50 after three periods, and so on. It should be noted that the multipliers shown in Table VI for each period are somewhat smaller than their counterparts in Table V, except for the first two periods; this is due to restraining influence of the monetary sector on the expansion process in Model IIIA, a phenomenon which is not present in Model IIA. The multipliers calculated in this way, of course, eventually equal the static multipliers appropriate to the underlying model (2.00 in the case of Model IIIA, for example).

This concept of the multiplier differs from the *static equilibrium multipliers* expressed in equation (2.11) or equation (3.17). The multiplier expressions in those equations give the change in *equilibrium* income which would result from a sustained change in the level of government purchases in each case. The multiplier values shown in Tables V and VI—and also in Table VII below—on the other hand, represent a dynamic or disequilibrium view of the multiplier. Rather than allowing the system to reestablish equilibrium, we have chosen to relate the amount by which income differs from its original equilibrium value at the end of any arbitrarily selected number of periods to the original spending change. Each of the expressions in the row labeled $\Delta Y_t / \Delta G^*$ in Tables V and VI—or Table VII—is a multiplier. Since the multiplier process has not been allowed to work itself out completely, however, any such multiplier is sometimes called a "truncated" multiplier. When the dynamic process is fully worked out and equilibrium income is reestablished, the change in equilibrium income will be that predicted by the comparable static multiplier equation. (This would be equation (3.17). But notice that, if we assume $v = 0$, which is the essential difference between Model IIA and Model IIIA, then the comparable static multiplier equation will be (2.11).) Thus the truncated multiplier approaches the static equilibrium multiplier in value over time. Both of these versions of the multiplier are consistent with the general definition of the multiplier as being the ratio of the change in income to the spending change which induced it.

TABLE VII

Changes in Income and Corresponding Values of the Truncated Dynamic Multipliers for Models IIA† and IIIA, Based on a Continuing Expenditure Change of ΔG^*

(time period [t])

	1	2	3	4	5	....n....
	0	ΔG^*	$\left[c(1-x)-\frac{vk}{m}\right]\Delta G^*$	$\left[c(1-x)-\frac{vk}{m}\right]^2\Delta G^*$	$\left[c(1-x)-\frac{vk}{m}\right]^3\Delta G^*$	$\left[c(1-x)-\frac{vk}{m}\right]^{n-2}\Delta G^*$
			ΔG^*	$\left[c(1-x)-\frac{vk}{m}\right]\Delta G^*$	$\left[c(1-x)-\frac{vk}{m}\right]^2\Delta G^*$	$\left[c(1-x)-\frac{vk}{m}\right]^{n-3}\Delta G^*$
				ΔG^*	$\left[c(1-x)-\frac{vk}{m}\right]\Delta G^*$	$\left[c(1-x)-\frac{vk}{m}\right]^{n-4}\Delta G^*$
					ΔG^*	$\left[c(1-x)-\frac{vk}{m}\right]^{n-5}\Delta G^*$
						ΔG^*
ΔY_t‡	0	ΔG^*	$\sum_{r=0}^{1}\left[c(1-x)-\frac{vk}{m}\right]^r\Delta G^*$	$\sum_{r=0}^{2}\left[c(1-x)-\frac{vk}{m}\right]^r\Delta G^*$	$\sum_{r=0}^{3}\left[c(1-x)-\frac{vk}{m}\right]^r\Delta G^*$	$\sum_{r=0}^{n-2}\left[c(1-x)-\frac{vk}{m}\right]^r\Delta G^*$
$\frac{\Delta Y_t}{\Delta G^*}$‡	0	1	$\sum_{r=0}^{1}\left[c(1-x)-\frac{vk}{m}\right]^r$	$\sum_{r=0}^{2}\left[c(1-x)-\frac{vk}{m}\right]$	$\sum_{r=0}^{3}\left[c(1-x)-\frac{vk}{m}\right]^r$	$\sum_{r=0}^{n-2}\left[c(1-x)-\frac{vk}{m}\right]$

†The basic difference between Model IIA and Model IIIA is that, in the former, the interest sensitivity of the demand for investment, v, is zero; thus there is no link between the monetary sector and the rest of the model, and the monetary sector cannot affect expenditure decisions and hence is omitted. In this table, the expressions as written are based on Model IIIA, but if v, the interest sensitivity of investment demand, is set equal to zero, the expressions will reflect the properties of Model IIA.

‡In this table, for convenience, we employ the standard notation for the sum of a series of terms having a variable in common. To take a simple example, suppose we wished to write the sum $Z_1 + Z_2 + Z_3 + Z_4 + Z_5$ in an abbreviated way. It is conventional to write this as $\sum_{r=1}^{5} Z_r$. This is read: "The summation of Z_r, with r taking on values from one to five." The symbol Σ is the *summation sign* and is the Greek letter sigma. In this expression, the subscript r provides a convenient way of handling the "length" of this sum—i.e., the fact that it includes all Z's from Z_1 to Z_5. In the expression

$$\sum_{r=0}^{n-2}\left[c(1-x)-\frac{vk}{m}\right]^r,$$

r is used to represent a variable exponent; that is, this expression represents the sum

$$\left[c(1-x)-\frac{vk}{m}\right]^0+\left[c(1-x)-\frac{vk}{m}\right]^1+\ldots+\left[c(1-x)-\frac{vk}{m}\right]^{n-3}+\left[c(1-x)-\frac{vk}{m}\right]^{n-2}$$

or, since any number raised to the zero power is unity,

$$1+\left[c(1-x)-\frac{vk}{m}\right]+\ldots+\left[c(1-x)-\frac{vk}{m}\right]^{n-3}+\left[c(1-x)-\frac{vk}{m}\right]^{n-2}.$$

The general expressions for the induced increase in income above the initial equilibrium level in any period, ΔY_t, and for the truncated multiplier for any period, $\Delta Y_t/\Delta G^*$, are developed in Table VII. In using this table, it is important to understand that the ΔY_t for any period is found by adding all of the items in the column corresponding to that period—that is, by adding *vertically* down a column. To find the value of the multiplier in the new equilibrium, we must evaluate a sum such as is found in the lower right-hand corner of Table VII, except that we must allow an infinite amount of time to pass. That is, we consider the value of the sum:

$$\frac{\Delta Y}{\Delta G^*} = \lim_{t \to \infty} \sum_{r=0}^{t-2} \left[c(1-x) - \frac{vk}{m}\right]^r .$$

This sum is found in the following way:

a) Consider the partial sum

$$\frac{\Delta Y_t}{\Delta G^*} = 1 + \left[c(1-x) - \frac{vk}{m}\right] + \left[c(1-x) - \frac{vk}{m}\right]^2 + \ldots \quad + \left[c(1-x) - \frac{vk}{m}\right]^{t-2} .$$

b) Multiply both sides of this expression by the term $-\left[c(1-x) - \frac{vk}{m}\right]$:

$$-\left[c(1-x) - \frac{vk}{m}\right]\frac{\Delta Y_t}{\Delta G^*} = -\left[c(1-x) - \frac{vk}{m}\right] - \left[c(1-x) - \frac{vk}{m}\right]^2 - \ldots$$
$$-\left[c(1-x) - \frac{vk}{m}\right]^{t-2} - \left[c(1-x) - \frac{vk}{m}\right]^{t-1}$$

c) Add these two equations (note that all the terms on the right-hand side cancel out except the first term of *(a)* and the last term of *(b)*):

$$\frac{\Delta Y_t}{\Delta G^*} - \left[c(1-x) - \frac{vk}{m}\right] \frac{\Delta Y_t}{\Delta G^*} = 1 - \left[c(1-x) - \frac{vk}{m}\right]^{t-1} .$$

d) Factor $\Delta Y_t / \Delta G^*$ out of the left-hand side of this equation, and multiply both sides by $1/\left[1 - c(1-x) + \frac{vk}{m}\right]$:

$$\frac{\Delta Y_t}{\Delta G^*} = \frac{1}{1 - c(1-x) + \frac{vk}{m}} \left\{ 1 - \left[c(1-x) - \frac{vk}{m}\right]^{t-1} \right\} .$$

(Note: this is the general expression for the truncated multiplier.)

e) Let t approach infinity; then $\left[c(1-x) - \frac{vk}{m}\right]^{t-1}$ approaches zero, $1 - \left[c(1-x) - \frac{vk}{m}\right]^{t-1}$ approaches unity, and $\Delta Y_t / \Delta G^*$ approaches the static equilibrium value $\Delta Y / \Delta G^*$. Thus, we obtain:

$$\frac{\Delta Y}{\Delta G^*} = \frac{1}{1 - c(1-x) + \frac{vk}{m}} .$$

We have shown that the truncated multiplier approaches the static equilibrium multiplier as the number of periods which have passed since the initial injection of spending becomes very large, just as was demonstrated for the specific numerical examples in Tables V and VI. In the present case, also, the truncated multiplier is zero for the first period and unity in the second period; after that its value rises steadily until it approaches the limiting value represented by the static equilibrium multiplier.

If the potential impact of a change in government spending on income is being studied, interest is focused on the resulting income level within a year, or some other relatively short period of time; it is not very useful to know what the new equilibrium level of income will be, since that level will not occur until an infinite amount of time has passed. Thus, the speed with which the multiplier process works is a matter of great practical significance, and this speed, measured in terms of calendar time, can be said to depend on two factors:

1. The number of unit time periods required to achieve some specified portion of the total ultimate effect; and
2. The length of a unit time period expressed in weeks or months.

The first of these factors, the number of time periods required, depends upon all of the marginal propensities and sensitivities in the multiplier expression—the marginal propensity to consume out of disposable income (c), the marginal propensity to pay taxes out of GNP (x), the marginal sensitivity of investment spending to the rate of interest (v), the marginal sensitivity of the demand for money balances to income (k), and the marginal sensitivity of the demand for money balances to the rate of interest (m). (If the model contained a supply-of-money relationship, instead of treating the money supply as fixed by the government, the number of periods required would also depend on the marginal sensitivity of the supply of money balances to the interest rate (e).) The nature of this dependence is illustrated in Table VIII by a number of numerical examples involving various values of the marginal propensity to consume and the marginal propensity to pay taxes. We are omitting the monetary sector in this illustration (assuming, in effect, that $v = 0$) as a means of keeping the presentation relatively simple, so that the calculations in Table VIII are based on Model IIA. The table is divided into four main sections, one each for marginal propensities to consume of 90 percent, 80 percent, 70 percent, and 60 percent. For each value of the marginal propensity to consume, multiplier calculations are presented for marginal propensities to pay taxes of 20 percent, 30 percent, and 40 percent. The values of the static equilibrium multiplier, as shown in column 2, decline as the marginal propensity to consume declines and as the marginal propensity to pay taxes increases. The same is true of the truncated multiplier calculated after five periods, as shown in column 3. It is interesting to note, however, that large multipliers work *relatively* more slowly than do smaller multipliers. Thus, while larger static multipliers (column 2) are associated with larger multipliers after five periods (column 3), the

TABLE VIII

The Speed of the Multiplier for Various Combinations of the Marginal Propensity to Consume Out of Disposable Income (*c*) and the Marginal Propensity to Collect Taxes Out of Total Income (*x*), Based on Model IIA†

	1	2	3	4	5
	Marginal Propensity to Collect Taxes Out of Income (x)	*Static Multiplier Value*	*Value of Truncated Multiplier at End of Fifth Period‡*	*Percent of Total Income Change Achieved by End of Fifth Period (3) ÷ (2) × 100*	*Number of Periods Needed to Achieve at Least 90% of Total Income Change§*
$c = .9$					
	.2	3.57	2.61	73.1%	9
	.3	2.70	2.27	84.1	6
	.4	2.17	1.99	91.7	5
$c = .8$					
	.2	2.78	2.31	83.1	7
	.3	2.27	2.05	90.3	5
	.4	1.92	1.82	94.7	5
$c = .7$					
	.2	2.27	2.05	90.3	5
	.3	1.96	1.85	94.4	5
	.4	1.72	1.67	97.1	4
$c = .6$					
	.2	1.92	1.82	94.8	5
	.3	1.72	1.67	97.1	4
	.4	1.56	1.53	98.3	4

†In this table, the speed of the multiplier is measured in terms only of the number of periods needed to achieve 90 percent of the total change in income, and the change in income achieved by the end of the fifth period. The period referred to is the period required to adjust production to sales.

‡This value is computed from the formula

$$\frac{\Delta Y}{\Delta G^*} = \frac{1}{1 - c(1-x)}\left\{1 - [c(1-x)]^{t-1}\right\}$$

when t = 5. This is the general expression for the truncated multiplier for model IIA.

§This value is calculated through the use of an expression for the ratio $\frac{\Delta Y_t}{\Delta Y}$, which can be found from the general expressions for the truncated multiplier and the static multiplier given on p. 35 (assuming $v = 0$) as follows:

$$\frac{\Delta Y_t}{\Delta Y} = \frac{\Delta Y_t/\Delta G^*}{\Delta Y/\Delta G^*} = \frac{\{1 - [c(1-x)]^{t-1}\}/[1 - c(1-x)]}{1/[1 - c(1-x)]}$$

$$= 1 - [c(1-x)]^{t-1}.$$

To find the number of periods needed to achieve 90 percent of the total change in income, set $\frac{\Delta Y_t}{\Delta Y}$ equal to 0.9 and solve for t:

$$0.9 = 1 - [c(1-x)]^{t-1},$$

so

$$[c(1-x)]^{t-1} = .1.$$

Using logarithms, we have

$$(t-1)\log[c(1-x)] = \log 0.1;$$
$$t \log[c(1-x)] = \log 0.1 + \log[c(1-x)]$$

and

$$t = \frac{\log 0.1 + \log[c(1-x)]}{\log[c(1-x)]}$$

percentage of the total multiplier effect that is achieved at the end of five periods is smaller for large than for small multipliers, as is shown in column 4. Column 5 brings out this same tendency in a different way: it shows that the number of time periods needed to achieve 90 percent of the total static multiplier effect is larger for large than for small multipliers.

The second factor determining the speed of the multiplier, the length of a unit multiplier time period, is the subject of the Ackley article included in this chapter. The article shows that the effects produced in a given "round" of multiplier expansion are spread out over time rather than all occurring at once and that the lags differ according to the type of expenditure involved. Thus, the time period is best viewed as a kind of average. Moreover, the time period varies according to the reaction times of businessmen in adjusting inventories and production rates. The article is analytical rather than empirical—that is, it makes no real effort to estimate the actual length of the period. Indeed, there has been little or no significant empirical research on the multiplier time period.

It may be useful to indicate at least in a crude way the probable dimensions of the speed of the multiplier. If we take the time period to be one quarter of a year, an estimate that seems not unreasonable based on the Ackley article, and take the marginal propensity to consume (c) to be 90 percent and the marginal propensity to pay taxes (x) to be 40 percent, values which are fairly realistic, the static multiplier is 2.17, and the multiplier applicable after five periods, or 15 months is 1.99. Thus, according to this estimate, a sustained increase of $10 billion in the annual rate of government purchases should raise GNP, expressed at annual rates, by about $20 billion after five quarters or 15 months.

Attention should perhaps be called explicitly to four important assumptions underlying this discussion. First, as we have already pointed out, no allowance is made in the model used to analyze the speed of the multiplier for the restraining effect of the monetary sector. We have assumed that this effect is inoperative; that is, we suppose that the real and monetary sectors of the model are unrelated, or, alternatively, that the Federal Reserve supplies enough bank reserves to enable the money supply to meet the rising demand for money associated with increasing GNP without any increase in interest rates. If these assumptions are erroneous, the expansion will produce a "feedback" effect which will push up interest rates. This will reduce the size of the static multiplier, as we already know. Using the analysis on which Table VIII is based, we can also infer the effects on the speed of the multiplier of changes in the size of the various monetary sensitivities. The larger the interest sensitivities of the demand for money and the supply of money, the larger the static multiplier but the slower the multiplier process (in the sense represented by the calculations in Table VIII). In other words, increases in m and e have the same effect on multiplier speed as increases in c. On the other hand, increases in the interest sensitivity of investment, v, or in the income sensitivity of the demand for money, k, have similar effects on the speed of the multiplier. The larger is either of these, the smaller the static multiplier value, but the more rapid the multiplier process.

The second assumption we have made is that there are sufficient unutilized resources to enable the expansion to take place without any appreciable effect on the price level. This assumption was emphasized earlier, in our comparison of Neoclassical and Keynesian views on monetary and fiscal policy, but it deserves repeating here. The situation will be different if the economy is operating close to full employment so that an increase in government spending will have its main effect on prices and only a moderate impact on output and employment. Third, no allowance is made for the possibility—indeed, probability—that businessmen will attempt to maintain or build up their inventories in the course of the expansion. If this were allowed for, it would undoubtedly raise the multipliers somewhat above the levels indicated.[19] Fourth, no consideration is given—either in this example or elsewhere in our discussion—to the possibility that an expansion of the multiplier type may induce an increase in private investment. This is, however, a likely possibility because the expanded sales will increase business profits and because the expanded production will increase the extent of utilization of existing plant and equipment. If such an increase in investment does occur, it is likely to add another important dimension to the expansion—in effect, raising the multiplier well above the levels we have been discussing. However, we have not attempted to include this possibility because, while the response of consumption to income changes is reasonably predictable, the investment response is a good deal less dependable, less well understood, and more likely to depend in a major way on the circumstances in which the expansion occurs.[20]

Some quantitative estimates of the size of the multiplier can be obtained from econometric models of the U.S. economy. One such model has been constructed and progressively refined by the Research Seminar in Quantitative Economics at the University of Michigan. This model is quite detailed, containing several dozen behavioral equations; however, it is basically similar to Model II, having equations to describe the behavior of households, firms, and the government sector. The coefficients are estimated

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[19]Actually the introduction of inventory investment complicates the analysis considerably. Since in full equilibrium inventories would presumably have reached the level in relation to sales that businessmen desired, there would, under these conditions, be no further additions to inventories. For this reason, inventory investment would have no effect on the static equilibrium multiplier. However, it would greatly affect the time path of movement from one equilibrium position to another; indeed, inventory investment may introduce a self-generating cycle in economic activity. In any case it would increase the effective truncated multipliers for the early stages of the expansion. See Lloyd A. Metzler, "The Nature and Stability of Inventory Cycles," *Review of Economic Statistics*, Vol. XXIII (August 1941), pp. 113–29.

[20]For a discussion of the multiplier effects of tax reduction, including probable effects on investment, see the selection entitled "The Effects of Tax Reduction on Output and Employment," from the *Annual Report of the Council of Economic Advisers*, January 1963, reprinted in Chapter 4 of this book.

statistically on the basis of past behavior of the variables.[21] Some of the typical multiplier values yielded by this model, based on the latest available estimates of its coefficients, are as follows:[22]

	Increase of $1 Billion in:		
	Government Purchases of Goods and Services	*Private Investment in Plant and Equipment*	*Federal Income Tax Collections*
Effect on GNP in first year........	$1.4 billion	$2.0 billion	$1.2 billion

This model, like all such econometric models of the economy, is dynamic in nature: that is, a change in (say) government purchases will set off a sequence of adjustments. However, it is important to note that the time unit used in such models is not an analytical multiplier time period but is rather some arbitrary unit of calendar time, usually a quarter or a year. The reason for this, of course, is that empirical observations on the variables are not available for such an essentially unmeasurable and irregular period as the multiplier time period. The model referred to above is based on annual observations on the variables, and the multipliers in the above table relate to the estimated effects in the first year.

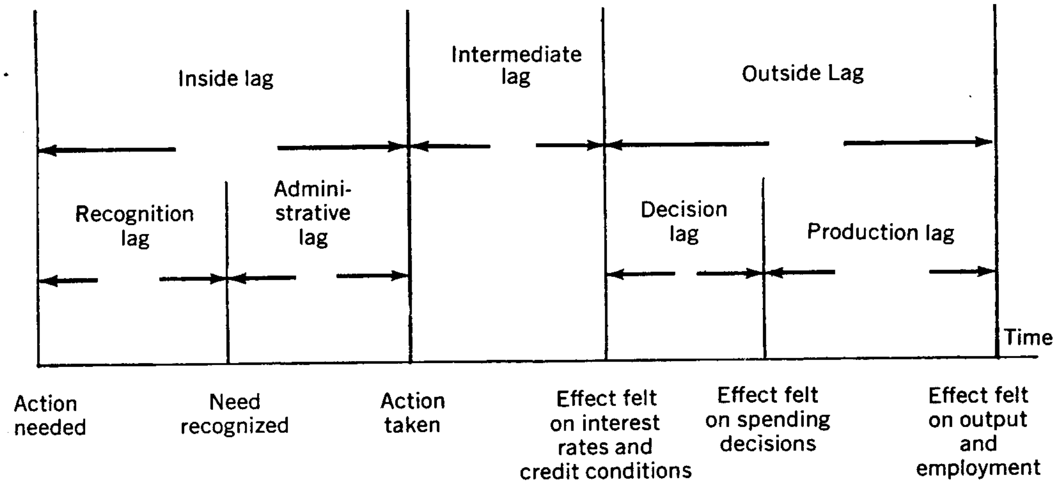
(b) Lags Associated with Adjustments in Monetary and Fiscal Policy

In addition to the lags involved in the working of the multiplier discussed above, there are further lags related to adjustments in monetary and fiscal policy that need to be considered. These lags are presented schematically in the following diagram, which relates specifically to monetary policy.

The process is assumed to start with a change in the economic situation which calls for some adjustment in monetary policy. For example, there might be a slowdown in the expansion of GNP and a rise in unemployment

[21] For a description of the way in which this model is derived and a detailed review of its structure, see Daniel B. Suits, "Forecasting and Analysis with an Econometric Model," *American Economic Review*, Vol. LII (March 1962), pp. 104 -32. A lengthier treatment is found in Daniels B. Suits, *The Theory and Application of Econometric Models* (Athens: Center of Economic Research, 1963). This book contains an entire chapter on policy multipliers in the U.S. economy. Information on the values of these multipliers can also be found in Daniel B. Suits, "Econometric Analysis of Disarmament Impacts," in Emile Benoit and Kenneth E. Boulding (eds.), *Disarmament and the Economy* (New York: Harper & Row, 1963), pp. 99 - 111. Finally, information on each year's estimate of the coefficients of this model is contained in the annual issues of the *Papers and Proceedings* of the Conference on the Economic Outlook held each year at the University of Michigan.

[22] In this model, the multiplier for investment spending is greater than for government expenditure, because investment expenditure is assumed to induce further inventory investment on the part of business, while government spending is not assumed to have such an effect.



which calls for a shift toward a more expansionary Federal Reserve policy. The time when this occurs is indicated by the caption "action needed," at the left-hand end of the time scale.

Three main elements in the lag in monetary policy adjustments can be distinguished:

1. The *inside lag*. This is the lag within the Federal Reserve System between the time action is needed and the time the action is actually taken. This lag can be broken into two subdivisions: (*a*) the *recognition lag* between the time action is needed and the time the need is recognized by the Federal Reserve authorities, and (*b*) the *administrative lag* between the time the need for action is recognized and the time the action (such as open market purchases of U.S. government securities) is actually taken. The length of the recognition lag presumably depends on the efficiency of the Federal Reserve in collecting and interpreting data relating to economic conditions. As a result of the organizational independence and flexibility of the Federal Reserve System, the administrative lag is presumably very short. These matters are discussed in several of the selections in Chapters 3 and 5.

2. The *intermediate lag*. This is the lag between the time the Federal Reserve takes action and the time the action produces a sufficient effect on interest rates (and other credit terms) to influence spending decisions significantly. The length of this lag depends on the behavior of commercial banks and other financial institutions and the functioning of financial markets—matters which are discussed in the selections in Chapters 2 and 5.

3. The *outside lag*. This is the lag between the change in interest rates (and credit conditions) and the initial impact on production and employment. This lag can be subdivided into two parts: (*a*) the *decision lag* between the change in interest rates and the change in spending decisions, and (*b*) the *production lag* between changes in spending decisions and the related initial changes in production and employment. It should be noted

that the production lag referred to here is in principle the same lag between changes in sales and changes in production that formed the basis for our earlier discussion of the multiplier time period. However, in this case, we are not discussing the full cumulative multiplier effects but merely the "first round" effects of the change in policy. After the effects discussed here had occurred, the multiplier process would take over and produce further effects not here taken into account.

The selection by Hamburger in Chapter 5 discusses the lags in monetary policy, paying particular attention to the production lag. The analysis includes investment in new housing and other consumer durables as well as business investment in inventories and in plant and equipment, and addresses itself to the magnitude of the effects of changes in interest rates on investment decisions in these various categories as well as to the lags in the appearance of these effects. It should be pointed out that there has been some controversy concerning the lags in monetary policy. It is an area in which research is just beginning, and the findings summarized by Hamburger should be taken, in general, as preliminary and suggestive rather than as in any sense conclusive.

For fiscal policy, the recognition lag is likely to be about the same as for monetary policy, since there is no reason to suppose that the economic intelligence apparatus of the authorities responsible for fiscal policy is either more or less efficient than that of the monetary authorities. However, the administrative lag for fiscal policy is likely to be much longer than that for monetary policy. This is especially true for tax adjustments, which ordinarily require a long (and uncertain) process of executive recommendation and legislative action. Changes in government expenditures may also require legislative action; however, even if all that is involved is a speedup of expenditures on projects that have already been approved by Congress, substantial time is likely to be needed to prepare plans and activate projects. All in all, the inside lag is likely to be much longer for fiscal than for monetary policy. On the other hand, the intermediate lag—which, in the case of fiscal policy, is the lag between the time when action is taken and the time when income or spending is affected—is likely to be much shorter for fiscal than for monetary policy. The decision lag also may be short—indeed, there is no such lag for changes in government purchases of goods and services. Since the production lag is in principle no different for fiscal than for monetary policy—although it may depend on the kind of expenditures that are involved—the outside lag is likely to be shorter for fiscal policy. To summarize:

Inside lag............................longer for fiscal policy
Intermediate lag....................shorter for fiscal policy
Outside lag...........................shorter for fiscal policy

Because of the much greater length of the administrative component of the inside lag, the overall lag for fiscal policy may frequently be longer than for monetary policy. Note, however, that the long administrative lag is not inherent in fiscal policy but is capable of being shortened greatly by changes in administrative arrangements. If this could be done—for example, by giving the President some authority to make countercyclical adjustments in tax rates, as recommended in the reading in Chapter 4 by the Council of Economic Advisers entitled "Formulating Fiscal Policy"—fiscal policy might become more quick-acting than monetary policy.

A discussion of the lags involved in the use of fiscal policy is presented in the selection by Albert Ando and E. Cary Brown in Chapter 4, and an interesting study of the lags involved in a particular episode—the 1964 tax reduction—is presented by Arthur Okun in his paper in the same chapter.